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$$(3) \quad c_n p^n + c_{n-1} p^{n-1} + \dots + c_1 p + c_0 + \frac{c_n + c_{n-1} + \dots + c_1 + c_0}{p-2} \\ = r_v p^v + r_{v-1} p^{v-1} + \dots + r_1 p + r_0 + \frac{n}{p-2},$$

from which the values of the c 's (and hence by (1) the value or values of m) may be found.

As an example, take the following case: $p=5$ and $s=3725$. We have from (3),

$$c_n p^n + c_{n-1} p^{n-1} + \dots + c_1 p + c_0 + \frac{c_n + c_{n-1} + \dots + c_1 + c_0}{p-2} \\ = 5^5 + 2 \cdot 5^4 + 4 \cdot 5^3 + 3 \cdot 5^2 + 3 \cdot 5 + 1 + \frac{3}{5}.$$

It is easy to show that $n=5$ and $c_5=1$; then that $c_4=2$, and $c_3=4$, and $c_2=3$, and $c_1=2$, and $c_0=2$. (Notice that the values of the c 's are most easily determined in the order given.) This gives

$$m=5^5 + 2 \cdot 5^4 + 4 \cdot 5^3 + 3 \cdot 5^2 + 3 \cdot 5 + 2 = 4962.$$

It is now evident that the solution or solutions may *always* be readily obtained from (3).

When $s=5$ and $p=3$, $m=7$ or 9 ; and this is sufficient to show that more than one solution exists in some cases even when p is odd.

An incomplete solution was received from Professor Zerr.

143. Proposed by JOHN D. WILLIAMS (being the first of 14 challenge problems published in 1832).

Make $x^2 + y^2 = a^2 = z^2 + w^2$ and $x^2 - w^2 = z^2 - y^2 = \square$.

I. Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let $x=2q(7q-5p)$, $y=25p^2-70pq+48q^2$,

$z=24p^2-70pq+50q^2$, $w=p(10q-7p)$.

$\therefore x^2 + y^2 = z^2 + w^2 = (25p^2 - 70pq + 50q^2)^2 = a^2$.

$x^2 - w^2 = z^2 - y^2 = 196q^4 - 280pq^3 + 140p^3q - 49p^4 = (14q^2 - 10pq - \frac{7}{4}p^2)^2$

when $q = \frac{1513p}{1680}$; $m = \frac{1513}{1680}$, where $pm = q$.

$\therefore x = \frac{473569p^2}{201600}$, $y = \frac{52319p^2}{58800}$, $z = \frac{85345p^2}{56448}$, $w = \frac{337p^2}{168}$.

Reducing to a common denominator and expunging common factors, we get

$x = 3314983p^2$, $y = 1255656p^2$, $z = 2133625p^2$, $w = 2830800p^2$.

$\therefore x^2 + y^2 = z^2 + w^2 = a^2 = (3544825p^2)^2$,

$x^2 - w^2 = z^2 - y^2 = b^2 = (1725017p^2)^2$.

Also $a - x = 2(339p)^2 = 2c^2$, $a - z = 2(840p)^2 = 2d^2$,

$a - y = (1513p)^2 = h^2$, $a - w = (845p)^2 = k^2$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

General expressions for the sides of duplicate right-triangles having the same hypotenuse are

$$x = (p^2 - q^2)(r^2 - s^2) + 4pqrs, \quad y = 2rs(p^2 - q^2) - 2pq(r^2 - s^2),$$

$$z = (r^2 - s^2)(p^2 + q^2), \quad w = 2rs(p^2 + q^2).$$

$$\therefore x^2 + y^2 = z^2 + w^2 = [(r^2 + s^2)(p^2 + q^2)]^2, \quad \text{Let } p=2, q=1.$$

$$\therefore x = 3(r^2 - s^2) + 8rs, \quad y = 6rs - 4(r^2 - s^2), \quad z = 5(r^2 - s^2), \quad w = 10rs.$$

$$x^2 + y^2 = z^2 + w^2 = [5(r^2 + s^2)]^2, \quad x^2 - w^2 = z^2 - y^2 = 9r^4 + 48r^3s - 54r^2s^2 - 48rs^3 + 9s^4 = \square. \quad \text{This is a square when } r = \frac{17}{2}s.$$

$$\therefore x = \frac{689s^2}{64}, \quad y = \frac{161s^2}{36}, \quad z = \frac{725s^2}{144}, \quad w = \frac{85s^2}{6}.$$

$$\therefore x = 2067s^2, \quad y = 644s^2, \quad z = 725s^2, \quad w = 2040s^2.$$

$$x^2 + y^2 = z^2 + w^2 = (2165s^2)^2 = a^2, \quad x^2 - w^2 = z^2 - y^2 = (333s^2)^2 = b^2.$$

A solution of this problem is given in J. D. Williams' *Algebra*, page 419. He starts with $a^2 = b^2 + f^2 = c^2 + e^2$, and $b^2 - c^2 = d^2 = e^2 - f^2$. Then he is to make $a^2 - b^2$ a square, $a^2 - c^2$ a square, and $b^2 - f^2$ a square. He assumes $a^2 = (p^2 + q^2)(r^2 + s^2)$, $b = pr \pm qs$, $c = ps \pm qr$. Then he assumes $r = pm - qn$, $s = pn + qm$. He finally arrives at the conclusion that $a = 697$, $b = 680$, $f = 158$, $c = 672$, $e = 185$, a set of erroneous values, as Dr. Zerr has pointed out. It is likely that Williams' solution may be carried out so that a set of correct values may be obtained. Williams proposed this problem in 1832 as a challenge problem to the mathematicians of the United States. Ed. F.

144. Proposed by JOHN D. WILLIAMS (being the ninth of his 14 challenge problems proposed in 1832).

Make $(m^2 + n^2)^2 x^2 \pm (m^2 + n^2)x = \square$, $(m^2 - n^2)^2 x^2 \pm (m^2 - n^2)x = \square$, and $4m^2 n^2 x^2 \pm 2mnx = \square$.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

$$\text{Let } m^2 + n^2 = p, \quad m^2 - n^2 = q, \quad 2mn = r.$$

$$\therefore p^2 x^2 \pm px = \square, \quad q^2 x^2 \pm qx = \square, \quad r^2 x^2 \pm rx = \square \dots (1, 2, 3).$$

$$\text{Let } p^2 x^2 + px = a^2 x^2; \quad \therefore x = p/(a^2 - p^2).$$

This value of x in (2) and (3) gives

$$q^2 p^2 + qp(a^2 - p^2) = \square, \quad r^2 p^2 + rp(a^2 - p^2) = \square \dots (4, 5).$$

$$\text{Let } q^2 p^2 + qp(a^2 - p^2) = [pq - b(a - p)]^2.$$

$$\therefore (a - p) = \frac{2pq(b + p)}{b^2 - pq}. \quad \text{This value of } a \text{ in (5) gives}$$

$$\begin{aligned} r^2 b^4 + 4b^3 pqr + 2b^3 pqr(2p + 2q - r) + 4bp^2 q^2 r + p^2 q^2 r^2 &= \square \\ &= (rb + 2bpq - pqr)^2, \quad \text{suppose.} \end{aligned}$$

$$\therefore b = \frac{2pqr}{pr + qr - pq}; \quad x = \frac{p}{a^2 - p^2} = \frac{(b^2 - pq)^2}{4bpq(b + p)(b + q)}.$$

$$\therefore x = \frac{-(pr + qr - pq)^2 - 4pqr^2}{8pqr(pr + qr - pq)(pq - pr + qr)(pq + pr - qr)}.$$